WES237B - Lab2

Quick Math Primer

Discrete Cosine Transform (DCT)

- 1D-DCT

$$F(u) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \Lambda(i).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] f(i)$$

$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

Discrete Cosine Transform (DCT)

- 1D-DCT

$$F = Cf$$

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- 2D-DCT

$$F(u,v) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \left(\frac{2}{M}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(i).\Lambda(j).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] cos\left[\frac{\pi.v}{2.M}(2j+1)\right].f(i,j)$$

$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

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- 2D-DCT

$$F(u,v) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \left(\frac{2}{M}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(i).\Lambda(j).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] cos\left[\frac{\pi.v}{2.M}(2j+1)\right] .f(i,j)$$

- apply 1D DCT (Vertically) to Columns
- apply 1D DCT (Horizontally) to resultant Vertical DCT above.
- or alternatively Horizontal to Vertical.

$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

Discrete Cosine Transform (DCT)

- 1D-DCT

$$F = Cf$$

$$F(u) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \Lambda(i).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] f(i)$$

$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

- 2D-DCT

$$F = CfC^T$$

$$F(u,v) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \left(\frac{2}{M}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(i).\Lambda(j).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] cos\left[\frac{\pi.v}{2.M}(2j+1)\right].f(i,j)$$

$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

Matrix Multiply

C = A * B

```
[110,  71, 146, 137]        [4, 5, 9, 2]     [9, 1, 7, 7]
[137,  68, 149, 140]   =    [7, 3, 9, 2]  *  [0, 3, 9, 9]
[ 45,  59, 113, 110]        [2, 7, 3, 3]     [8, 4, 7, 6]
[ 59,  88, 109, 106]        [3, 3, 3, 8]     [1, 8, 5, 5]
```

Matrix Multiply

C = A * B

$$\begin{bmatrix} \boxed{110} & 71 & 146 & 137 \\ 137 & 68 & 149 & 140 \\ 45 & 59 & 113 & 110 \\ 59 & 88 & 109 & 106 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 9 & 2 \\ 7 & 3 & 9 & 2 \\ 2 & 7 & 3 & 3 \\ 3 & 3 & 3 & 8 \end{bmatrix} * \begin{bmatrix} 9 & 1 & 7 & 7 \\ 0 & 3 & 9 & 9 \\ 8 & 4 & 7 & 6 \\ 1 & 8 & 5 & 5 \end{bmatrix}$$

$$c_{i,j} = \sum a_{i,k} b_{k,j}$$

110 = 4*9 + 5*0 + 9*8 + 2*1

71 = 4*1 + 5*3 + 9*4 + 2*8

Block Matrix Multiply

C = A * B

$$
\begin{array}{cc}
c1 & c2 \\
\boxed{\begin{matrix}110 & 71 \\ 137 & 68\end{matrix}} & \boxed{\begin{matrix}146 & 137 \\ 149 & 140\end{matrix}} \\
\boxed{\begin{matrix}45 & 59 \\ 59 & 88\end{matrix}} & \boxed{\begin{matrix}113 & 110 \\ 109 & 106\end{matrix}} \\
c3 & c4
\end{array}
=
\begin{array}{cc}
a1 & a2 \\
\boxed{\begin{matrix}4 & 5 \\ 7 & 3\end{matrix}} & \boxed{\begin{matrix}9 & 2 \\ 9 & 2\end{matrix}} \\
\boxed{\begin{matrix}2 & 7 \\ 3 & 3\end{matrix}} & \boxed{\begin{matrix}3 & 3 \\ 3 & 8\end{matrix}} \\
a3 & a4
\end{array}
*
\begin{array}{cc}
b1 & b2 \\
\boxed{\begin{matrix}9 & 1 \\ 0 & 3\end{matrix}} & \boxed{\begin{matrix}7 & 7 \\ 9 & 9\end{matrix}} \\
\boxed{\begin{matrix}8 & 4 \\ 1 & 8\end{matrix}} & \boxed{\begin{matrix}7 & 6 \\ 5 & 5\end{matrix}} \\
b3 & b4
\end{array}
$$

c1 = a1*b1 + a2*b3

Block Matrix Multiply

C = A * B

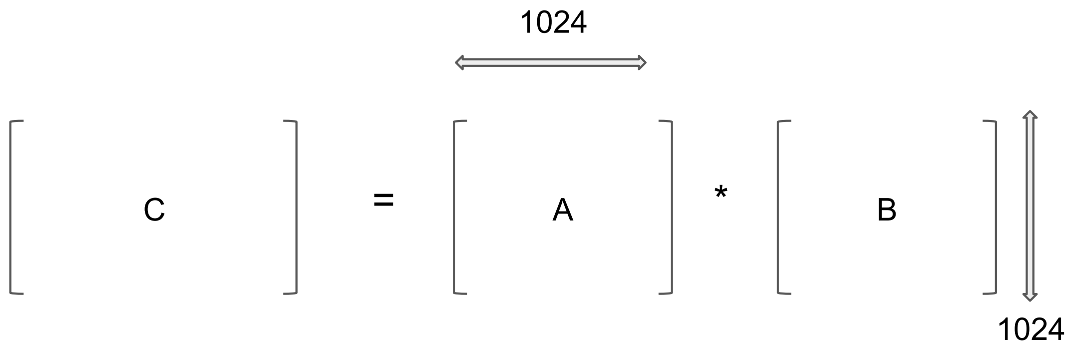
$$
\begin{array}{cccccccc}
c1 & c2 & & a1 & a2 & & b1 & b2 \\
\boxed{\begin{matrix}110 & 71 \\ 137 & 68\end{matrix}} & \boxed{\begin{matrix}146 & 137 \\ 149 & 140\end{matrix}} & & \boxed{\begin{matrix}4 & 5 \\ 7 & 3\end{matrix}} & \boxed{\begin{matrix}9 & 2 \\ 9 & 2\end{matrix}} & & \boxed{\begin{matrix}9 & 1 \\ 0 & 3\end{matrix}} & \boxed{\begin{matrix}7 & 7 \\ 9 & 9\end{matrix}} \\
 & & = & & & * & & \\
\boxed{\begin{matrix}45 & 59 \\ 59 & 88\end{matrix}} & \boxed{\begin{matrix}113 & 110 \\ 109 & 106\end{matrix}} & & \boxed{\begin{matrix}2 & 7 \\ 3 & 3\end{matrix}} & \boxed{\begin{matrix}3 & 3 \\ 3 & 8\end{matrix}} & & \boxed{\begin{matrix}8 & 4 \\ 1 & 8\end{matrix}} & \boxed{\begin{matrix}7 & 6 \\ 5 & 5\end{matrix}} \\
c3 & c4 & & a3 & a4 & & b3 & b4
\end{array}
$$

c1 = a1*b1 + a2*b3

c2 = a1*b2 + a2*b4

MM vs. BMM

Suppose cache size is 1024 8-bit parameters (8kb)



$$c_{i,j} = \sum a_{i,k} b_{k,j}$$

MM vs. BMM

Suppose cache size is 1024 8-bit parameters (8kb)

1024

$$\begin{bmatrix} \quad C \quad \end{bmatrix} = \begin{bmatrix} \quad A \quad \end{bmatrix} * \begin{bmatrix} \quad B \quad \end{bmatrix}$$

1024

$$c_{i,j} = \sum a_{i,k} b_{k,j}$$

c1 += a1*b1

c2 += a1*b2

16x16 blocks:
We keep 2x16x16 and C submatrices in cash

64 blocks

$$\begin{bmatrix} \quad C \quad \end{bmatrix} = \begin{bmatrix} \quad A \quad \end{bmatrix} * \begin{bmatrix} \quad B \quad \end{bmatrix}$$

64 blocks

MM vs. BMM

Suppose cache size is 1024 8-bit parameters (8kb)

1024

$$\begin{bmatrix} \quad C \quad \end{bmatrix} = \begin{bmatrix} \quad A \quad \end{bmatrix} * \begin{bmatrix} \quad B \quad \end{bmatrix}$$

1024

$$c_{i,j} = \sum a_{i,k} b_{k,j}$$

c1 += a2*b3

c2 += a2*b4

16x16 blocks:
We keep 2x16x16 and C submatrices in cash

64 blocks

$$\begin{bmatrix} \quad C \quad \end{bmatrix} = \begin{bmatrix} \quad A \quad \end{bmatrix} * \begin{bmatrix} \quad B \quad \end{bmatrix}$$

64 blocks